

Colored dark matter

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The idea is considered that a classical non-Abelian gauge field can be considered as a dark matter candidate. It is shown that Yang-Mills equations have solutions with such distribution of the mass density that allows to describe a rotational curve of spiral galaxies. The conditions necessary for such consideration are considered. One parameter is estimated from Yang-Mills equations and from astrophysical observations (stars rotation curve). The agreement is to within 1%.

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I. INTRODUCTION

The electromagnetic field behaves at different conditions either as pure quantum or almost as a classical field. But for non-Abelian gauge fields we know only quantum manifestations of these fields. Here we would like to present the idea that classical non-Abelian gauge fields can become visible as DM.

The problem of the DM nature is now one of the most fundamental problems in modern physics. The reason is to explain the invisibility of DM it is necessary to involve such kinds of an exotic matter as, for example, WIMP, supersymmetric particles etc.

In this article we offer a new model of DM. In the studied model the DM is an interior of an abortive singularity or black hole. We understand this suggestion as follows.

In Ref's [7] [8] it is shown that the energy density of spherically symmetric solutions of Yang-Mills equations may have non-standard behavior (behind an exception, of course, 't Hooft - Polyakov monopole). For example, in Ref. [8] it is shown that the energy density is weakly decreasing. It is natural that such distribution of the matter will give diverging solutions by switching on the gravity. For some boundary conditions at the center it will be an interior of a non-Abelian back hole and for other values of the boundary conditions a singularity located at some distance from the center will appear. But one very interesting subtlety exists here. This solutions are classical ones. The careful analysis shows that at the infinity these solutions are strongly oscillating in the space (gravity is switched off) and the period of the oscillations increases by moving away from the centre. It is obvious that on some distance from the center the quantum fluctuations of gauge field on the period distance of oscillations become comparable with a field magnitude. It means that at this distance the gauge field becomes quantum one. The problem in calculating of such distribution of the non-Abelian gauge field is that the gauge field becomes essentially non-perturbative and it is impossible to apply the perturbative Feynman diagram techniques to its description.

The idea presented here is that the radius where the gauge field becomes quantum can be less than the radius of a singular or an event horizon corresponding to a non-Abelian black hole. In this case this object does become neither a singularity nor a black hole (abortive singularity/black hole). It looks as follows: in space there is a sphere filled with a classical gauge field and on border of this sphere the non-Abelian gauge field becomes quantum one and the mass contained in this sphere is not enough for formation of a singularity or an event horizon. The color gauge field in the sphere does not interact with the elementary particles because the particles are colorless and consequently this classical field is invisible and can be applied as a candidate of DM.

In fact in this paper we consider the idea that in a non-Abelian gauge theory may exist such space distribution of a gauge field that classical and quantum phases exist simultaneously but spatially separated. In this case the jump condition from classical phase to quantum one is the strong oscillations of classical non-Abelian field. These oscillations leads to the fact that quantum fluctuations become essential at a distance of oscillation period. In other words the space is filled with a non-perturbative gauge vacuum where there exist defects filled with the classical gauge field. The galaxies are located in these defects and the classical gauge field is the DM.

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II. THE INTERIOR OF AN EINSTEIN-YANG-MILLS SINGULARITY/BLACK HOLE

In this section we would like to show that usually the $SU(3)$ gauge field distribution leads to a singular spacetime. We use the following metric

$$ds^2 = e^{\nu(r)} \left[1 - \frac{M(r)}{r} \right] dt^2 - \frac{dr^2}{1 - \frac{M(r)}{r}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (1)$$

where t, r, θ, φ are usual spherical coordinates. Substituting metric (1) and $SU(3)$ gauge potential (A1)-(A6) into Einstein-Yang-Mills equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}, \quad (2)$$

$$D_\nu F^{a\mu\nu} = 0, \quad (3)$$

$$T_{\mu\nu} = -F_{\mu\alpha}^a F_\nu^{a\alpha} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}^a F^{a\alpha\beta} \quad (4)$$

gives us Einstein equations

$$M' = \frac{\kappa}{r^2} \left\{ \frac{2}{3} \frac{e^{-\nu}}{1 - \frac{M}{r}} \left[6v^2 w^2 + \left(1 - \frac{M}{r} \right) (w - rw')^2 \right] + 2 \left[(1 - v^2)^2 + 2r^2 \left(1 - \frac{M}{r} \right) v'^2 \right] \right\}, \quad (5)$$

$$r \left(1 - \frac{M}{r} \right) \nu' = \frac{8\kappa}{r^2} \left[\frac{e^{-\nu}}{1 - \frac{M}{r}} v^2 w^2 + r^2 \left(1 - \frac{M}{r} \right) v'^2 \right], \quad (6)$$

$$-\frac{\nu''}{2} \left(1 - \frac{M}{r} \right) + \frac{M''}{2r} - \frac{\nu'^2}{4} \left(1 - \frac{M}{r} \right) + \frac{3}{4} \frac{M'\nu'}{r} - \frac{\nu'}{4r} \left(2 + \frac{M}{r} \right) = -\frac{2}{3} \frac{\kappa}{r^4} \left[3(1 - v^2)^2 + e^{-\nu} (w - rw')^2 \right] \quad (7)$$

and Yang-Mills equations

$$\left(1 - \frac{M}{r} \right) v'' + \left(-\frac{M'}{r} + \frac{M}{r^2} \right) v' = \frac{v}{r^2} (v^2 - 1) - \frac{e^{-\nu}}{r^2 \left(1 - \frac{M}{r} \right)} v w^2, \quad (8)$$

$$w'' - w'\nu' + w \frac{\nu'}{r} = \frac{6}{r^2 \left(1 - \frac{M}{r} \right)} w v^2 \quad (9)$$

where κ is the gravitational constant, $a = 1, 2, \dots, 8$ is the color index and $\chi(r) = h(r) = 0$. The analytical solution does not exist and we search the numerical solutions for this equations set.

We search for the solution inside of the non-Abelian singularity/black hole. It means that we should start the solution from the point $r = 0$. The analytical solution close to the center is

$$M(r) = M_3 \frac{r^3}{6} + \mathcal{O}(r^5), \quad \nu(r) = \nu_2 \frac{r^2}{2} + \mathcal{O}(r^4), \quad (10)$$

$$v(r) = 1 + v_2 \frac{r^2}{2} + \mathcal{O}(r^4), \quad w(r) = w_3 \frac{r^3}{6} + \mathcal{O}(r^5), \quad (11)$$

$$M_3 = 5\kappa v_2^2, \quad \nu_2 = 8\kappa v_2. \quad (12)$$

The numerical solution can not start from the point $r = 0$ since Eq's (5)-(7) have terms $1/r^2$. Consequently we should start from the point $x = \delta \ll r/\sqrt{\kappa}$. The boundary conditions are

$$M(\delta) = M_3 \frac{\delta^3}{6}, \quad \nu(\delta) = \nu_2 \frac{\delta^2}{2}, \quad (13)$$

$$v(\delta) = 1 + v_2 \frac{\delta^2}{2}, \quad v'(\delta) = v_2 \delta, \quad (14)$$

$$w(\delta) = w_3 \frac{\delta^3}{6}, \quad w'(\delta) = w_3 \frac{\delta^2}{2}. \quad (15)$$

We present the profiles of the functions $g^{rr} = (1 - \frac{M(r)}{r})$, $\nu(r)$, $v(r)$, $w(r)$ in Fig's 1 - 3. We see that at a point $r = r_H$

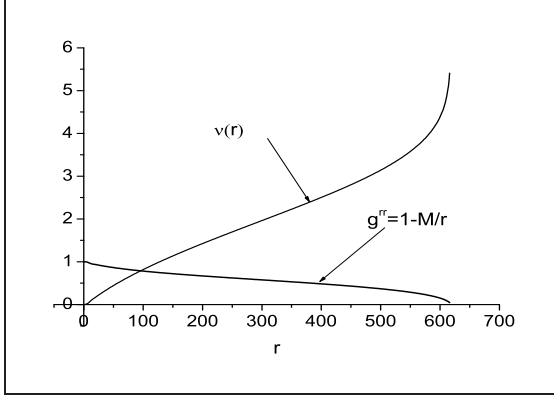


FIG. 1: The profile of the functions $g^{rr} = 1 - \frac{M(r)}{r}$ and $\nu(r)$.

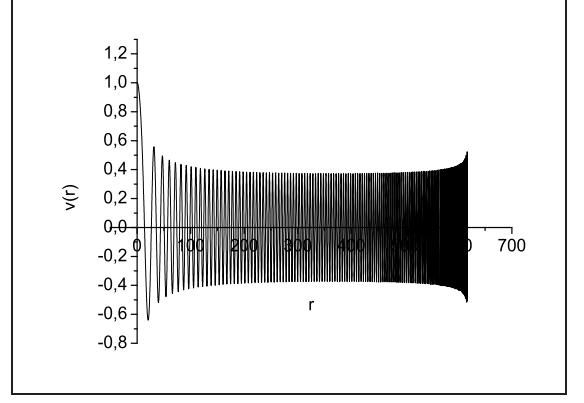


FIG. 2: The profile of the function $v(r)$.

the function $g^{rr}(r_H) = 1 - \frac{M(r_H)}{r_H} = 0$. The energy density is

$$\varepsilon = T_0^0 = -F_{0\alpha}F^{0\alpha} + \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} = \frac{2}{3g^2} \frac{e^{-\nu}}{r^4 \left(1 - \frac{M}{r}\right)} \left[6v^2w^2 + \left(1 - \frac{M}{r}\right) (w - rw')^2 \right] + \frac{2}{r^4} \left[(1 - v^2)^2 + 2r^2v'^2 \right] \quad (16)$$

and its profile in Fig. 4 is presented. Consequently the numerical analysis have shown us that at the point r_H where

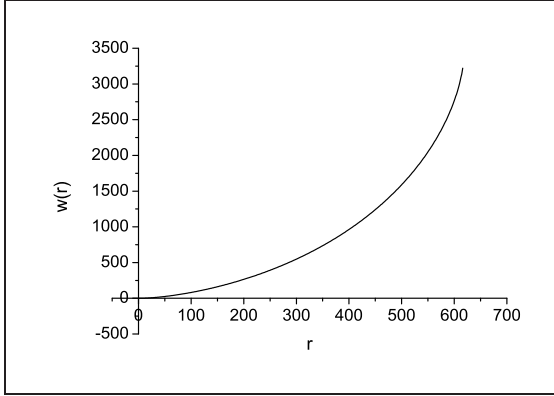


FIG. 3: The profile of the function $w(r)$.

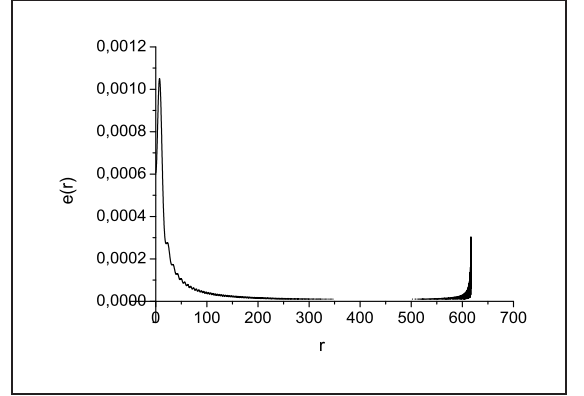


FIG. 4: The profile of the function energy density.

$1 - M(r_H)/r_H = 0$ there is a real or coordinate singularity (for some boundary conditions it will be an event horizon). From Fig. 2 we see that the function $v(r)$ is oscillating function and the period of oscillations increases with the moving away from the center. At some distance r_q from the center the quantum fluctuations on the distance of period will be comparable with the field magnitude. In this case the classical gauge field becomes quantum one. If it occurs on the distance $r_q < r_H$ then we will have a ball filled with the classical gauge field inside quantum gauge field. In Ref. [10] is claimed that the non-perturbative quantized SU(3) gauge field can be described using two scalar fields. These fields decrease very quickly (exponentially) to a ground state. If this so then the situation looks as follows: in the space there is a ball filled with the classical gauge field with weakly decreasing energy density. At some distance r_q from the center the field becomes quantum one and the energy density very quickly (exponentially) decreases to a ground state (non-perturbative vacuum). The idea presented here is that the ball can be considered as the DM as usual elementary particles are colorless and consequently can not interact with the color classical SU(3) gauge fields.

Let us note that non-Abelian black holes and particlelike solutions exist only with a special choice of the boundary conditions on the event horizon or at the center. We have to underline that in contrast with the non-Abelian black holes and particlelike solutions we consider the solutions with arbitrary boundary conditions.

In the following sections we will estimate the radius r_H of a real/cooordinate singularity and the radius r_q where the transition from the classical phase to quantum one occurs.

III. THE ESTIMATION OF THE RADIUS OF REAL OR COORDINATE SINGULARITY

Unfortunately the solution presented in previous section is numerical one that does not allow us to calculate the sphere where the above mentioned singularity (real or coordinate) is. We will estimate the radius of a singular sphere (using Newton gravity) in the following way. The radius is estimated as the place where the Newtonian gravitational potential becomes so strong that a test particle should have a velocity of light to get from this point on infinity. We test this method for the Schwazschild black hole solution. The energy conservation law tells us

$$\frac{mc^2}{2} - G \frac{mM}{r_H} = 0 \quad (17)$$

here G is the Newtonian constant; the first and second terms are kinetic and potential energies of a test particle with the mass m and M is the mass of a singularity or a black hole. As a result we have the radius of a singularity or an event horizon

$$r_H = \frac{2GM}{c^2} \quad (18)$$

that absolutely precisely coincides with the event horizon radius calculated in general relativity.

Now we will try to estimate the radius of the singularity or the event horizon for the classical distribution of the SU(3) gauge fields (A8)-(A11). We will work in Newton gravity because the values v_2 and w_3 are small enough, i.e. the magnitudes of the SU(3) gauge fields close to the center are small enough. In this case Yang-Mills equations in a flat space are

$$D_\nu F^{a\mu\nu} = 0 \quad (19)$$

and with ansatz (A8)-(A11) where $\chi(r) = h(r) = 0$ we have the following equations

$$x^2 w'' = 6wv^2, \quad (20)$$

$$x^2 v'' = v^3 - v - vw^2 \quad (21)$$

here the dimensionless radius $x = r/r_0$ is introduced and r_0 is an arbitrary constant. In fact Eq's (20) (21) are Eq's (8) (9) in Minkowski spacetime. The asymptotical behavior $x \gg 1$ of the solution is [8]

$$v(x) \approx A \sin(x^\alpha + \phi_0), \quad (22)$$

$$w(x) \approx \pm \left[\alpha x^\alpha + \frac{\alpha - 1}{4} \frac{\cos(2x^\alpha + 2\phi_0)}{x^\alpha} \right], \quad (23)$$

$$3A^2 = \alpha(\alpha - 1) \quad (24)$$

with $\alpha > 1$. The energy density $\epsilon(x)$ is

$$\epsilon(r) = -F_{0i}^a F^{a0i} + \frac{1}{4} F_{ij}^a F^{aij} = \frac{1}{g^2 r_0^4} \left[4 \frac{v'^2}{x^2} + \frac{2}{3} \frac{(xw' - w)^2}{x^4} + 2 \frac{(v^2 - 1)^2}{x^4} + 4 \frac{v^2 w^2}{x^4} \right] = \frac{1}{g^2 r_0^4} \epsilon(x) \quad (25)$$

where $v' = dv/dx, w' = dw/dx$. Asymptotically the dimensionless energy density is

$$\epsilon_\infty(x) \approx \frac{2}{3} \alpha^2 (\alpha - 1) (3\alpha - 1) \left(\frac{r}{r_0} \right)^{2\alpha - 4}. \quad (26)$$

The numerical analysis shows that the functions $v(r)$ and $w(r)$ quickly attain an asymptotic form. Therefore we will use (26) to estimate the mass $\mathcal{M}(r)$ under radius r

$$\mathcal{M}(r) = \frac{4\pi}{c^2} \int_0^r r^2 \epsilon(r) dr \approx \frac{8\pi}{3g^2 r_0 c^2} \frac{\alpha^2 (\alpha - 1) (3\alpha - 1)}{2\alpha - 1} \left(\frac{r}{r_0} \right)^{2\alpha - 1}. \quad (27)$$

The same calculations as in (17) gives us

$$\frac{mc^2}{2} - G \frac{m\mathcal{M}(r_H)}{r_H} = 0 \quad (28)$$

that leads to

$$r_H \approx r_0 \left[\frac{3}{4} g'^2 \frac{2\alpha - 1}{\alpha^2(\alpha - 1)(3\alpha - 1)} \left(\frac{r_0}{l_{Pl}} \right)^2 \right]^{\frac{1}{2\alpha - 2}} \quad (29)$$

where $\frac{1}{g'^2} = \frac{4\pi/g^2}{\hbar c}$ is the dimensionless coupling constant in the SU(3) gauge theory; $1/g$ is the analog of the electric charge in electrodynamics; $l_{Pl} = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-35}$ m is the Planck length. The numerical factor

$$\frac{3}{4} g'^2 \frac{2\alpha - 1}{\alpha^2(\alpha - 1)(3\alpha - 1)} \approx 1 \quad (30)$$

and consequently

$$r_H \approx r_0 \left(\frac{r_0}{l_{Pl}} \right)^{\frac{1}{\alpha - 1}}. \quad (31)$$

Using the parameters r_0 and α which are calculated in Appendixes B and C we have

$$r_H \approx 10^{28} \text{m} \quad (32)$$

that is much more than the galaxy radius.

Now we can define the natural choice of the parameter r_0 . At the origin Eq's (20) (21) have the solution that can be presented as series (11)

$$v(r) = 1 + \frac{1}{2} (r_0^2 v_2) \left(\frac{r}{r_0} \right)^2 + \mathcal{O} \left[\left(\frac{r}{r_0} \right)^4 \right] = 1 + v'_2 \frac{x^2}{2} + \mathcal{O}(x^4), \quad (33)$$

$$w(r) = \frac{1}{6} (r_0^3 w_3) \left(\frac{r}{r_0} \right)^3 + \mathcal{O} \left[\left(\frac{r}{r_0} \right)^5 \right] = w'_3 \frac{x^3}{6} + \mathcal{O}(x^5). \quad (34)$$

Therefore the natural choice of the parameter r_0 is

$$\text{either } r_0^2 = \frac{1}{v_2} \quad \text{or} \quad r_0^3 = \frac{1}{w_3}. \quad (35)$$

IV. THE TRANSITION FROM CLASSICAL PHASE TO QUANTUM ONE

The idea considered above mentioned solution of the classical Yang-Mills equations can not be extended up to infinity because in some place the space oscillations of the classical gauge field becomes so strong that non-perturbative quantum effects should be taken into account. In this section we try to estimate the radius where it can happen.

Following on the Heisenberg uncertainty principle

$$\frac{1}{c} \Delta F_{ti}^a \Delta A^{ai} \Delta V \approx \hbar \quad (36)$$

here ΔF_{ti}^a is a quantum fluctuation of color electric field F_{ti}^a ; ΔA^{ai} is a quantum fluctuation of color electric potential A^{ai} ; ΔV is the volume where the quantum fluctuations ΔF_{ti}^a and ΔA^{ai} takes place; $a = 1, 2 \dots 8$ is the color index; $i = 1, 2, 3$ is the space index.

For the ansatz (A8)-(A11)

$$F_{t\theta}^2 = -\frac{2}{g} \sin \theta \frac{vw}{r}. \quad (37)$$

We introduce the physical component of the $F_{t\theta}^2$

$$|\tilde{F}_{t\theta}^2| = \sqrt{F_{t\theta}^2 F_t^{2\theta}} = \frac{2}{g} \sin \theta \frac{vw}{r^2}. \quad (38)$$

To an accuracy of a numerical factor the fluctuations of the SU(3) color electric field are

$$\Delta \tilde{F}_{t\theta}^2 \approx \frac{1}{g} \frac{1}{r^2} (\Delta v \, w + v \, \Delta w). \quad (39)$$

For the ansatz (A8)-(A11)

$$A_\theta^2 = 0, \quad (40)$$

$$A_\theta^{1,3,4,6,8} \approx \frac{1}{g} v. \quad (41)$$

Introducing the physical components of the gauge potential $A_\theta^{1,3,4,6,8}$

$$\left| \tilde{A}_\theta^{1,3,4,6,8} \right| = \sqrt{A_\theta^{1,3,4,6,8} A^{1,3,4,6,8;\theta}} \approx \frac{1}{g} \sin \theta \frac{v}{r} \quad (42)$$

we assume that

$$\Delta \tilde{A}_\theta^2 \approx \Delta \tilde{A}_\theta^1 \approx \frac{1}{g} \sin \theta \frac{\Delta v}{r}. \quad (43)$$

The volume ΔV is

$$\Delta V = 4\pi r^2 \Delta r. \quad (44)$$

The period of space oscillations by $r \gg r_0$ can be defined in the following way

$$(x + \lambda)^\alpha - x^\alpha \approx \alpha \frac{\lambda}{x^{1-\alpha}} = 2\pi; \quad x = \frac{r}{r_0}. \quad (45)$$

We suppose that the place where the SU(3) classical color field becomes quantum one is defined as the place where the quantum fluctuations in the volume $\Delta V = 4\pi r^2 \Delta r$ with

$$\frac{\Delta r}{r_0} \approx \lambda \approx \frac{1}{\alpha} \frac{2\pi}{x^{\alpha-1}} \quad (46)$$

of the corresponding field becomes comparable with magnitude of these fields

$$\Delta v \approx v, \quad \Delta w \approx w \quad (47)$$

Substituting of Eq's (39), (22)-(24), (43), (44), (46) and (47) into Eq. (36) we obtain

$$\left(\frac{g'}{A} \right)^2 \approx 2\pi \quad (48)$$

where $\frac{1}{g'^2} = \frac{4\pi}{g^2 \hbar c}$ is the dimensionless coupling constant that is similar to the fine structure constant in quantum electrodynamics $\alpha = \frac{e^2}{\hbar c}$. In quantum chromodynamics $\beta = 1/g'^2 \geq 1$. If we choose $1/g' \approx 1$ and from Fig. 2 we take $A \approx 0.4$ we see that

$$\left(\frac{g'}{A} \right)^2 \approx 6.25 \quad (49)$$

that is comparable with $2\pi \approx 6.28$.

Thus in this section we have shown that if the condition (48) is true then at some distance from the center the transition from the classical phase to quantum one occurs. Unfortunately the rough estimation presented in this section does not allow us to calculate the radius where such transition takes place. For the exact evaluation of the place where such transition happens it is necessary to have *non-perturbative* quantization methods which are missing at the moment.

V. THE ROTATION CURVE OF YANG-MILLS COLORED DARK MATTER

In this section we would like to show that the solution of Eq's (20) (21) really has such mass density distribution that it is in a good agreement with the Universal Curve Rotation.

A. Numerical investigation

In this subsection we present the typical numerical solution of Eq's (20) (21). For the numerical investigation we have to start from the point $x = \delta \ll 1$. Here we have approximate solution (33) (34) and now we choose the parameter r_0 as

$$r_0 = \frac{1}{w_3^{1/3}}. \quad (50)$$

The typical behavior of functions $v(x)$ and $w(x)$ is presented in Fig. 5.

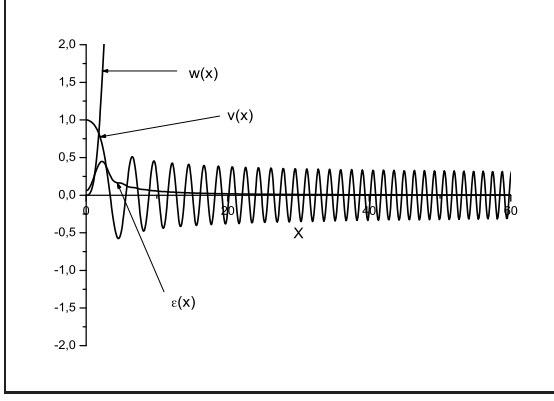


FIG. 5: The profile of functions $w(x), v(x), \varepsilon(x)$, $v_2 = -0.1$, $w_3 = 1$.

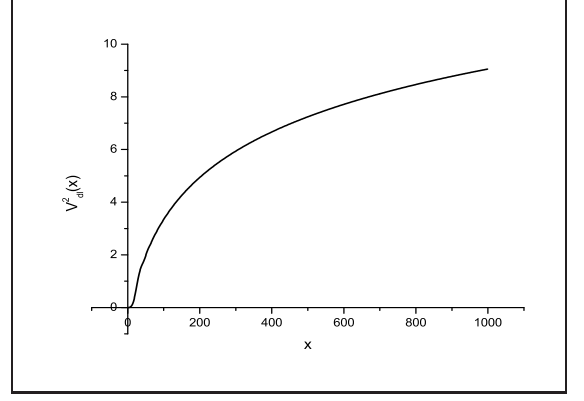


FIG. 6: The profile of the dimensionless rotation curve $V_{dl}^2 = g'^2 \left(\frac{r_0}{l_{Pl}} \right)^2 \frac{V^2(x)}{c^2}$.

The mass density $\rho(r)$ is

$$\rho(r) = \frac{1}{g^2 c^2 r_0^4} \rho(x) \quad (51)$$

where $\rho(x) = \varepsilon(x)$ and $\varepsilon(x)$ is given in Eq. (25). The profile of the dimensionless energy density $\varepsilon(x)$ in Fig. 6 is presented.

The rotation curve is defined as

$$V^2 = G \frac{\mathcal{M}(r)}{r} = 4\pi G \frac{1}{r} \int_0^r r'^2 \rho(r') dr' = \frac{G\hbar}{c^3} \frac{c^2}{g'^2 r_0^2} \frac{\mathcal{M}(x)}{x} = \left[\frac{1}{g'^2} \left(\frac{l_{Pl}}{r_0} \right)^2 \frac{\mathcal{M}(x)}{x} \right] c^2 \quad (52)$$

where $\mathcal{M}(x)$ is the dimensionless mass of the color fields A_μ^a inside the sphere of radius $r = xr_0$, $g'^2 = g^2 \hbar / 4\pi$ is the dimensionless coupling constant, G is the Newton gravitational constant. The parameter $\alpha \approx 1.31$ can be found using fitting of functions $w(x)$ or $\varepsilon(x)$, for details see Appendix B.

B. The comparison with a Universal Rotation Curve of spiral galaxies

In Ref. [11] a Universal Rotation Curve of spiral galaxies is offered that describes any rotation curve at any radius with a very small cosmic variance

$$V_{URC} \left(\frac{r}{R_{opt}} \right) = V(R_{opt}) \left[\left(0.72 + 0.44 \log \frac{L}{L_*} \right) \frac{1.97 X^{1.22}}{(X^2 + 0.782)^{1.43}} + 1.6 e^{-0.4(L/L_*)} \frac{X^2}{X^2 + 1.52 \left(\frac{L}{L_*} \right)^{0.4}} \right]^{1/2} \text{ km s}^{-1} \quad (53)$$

where $R_{opt} \equiv 3.2 R_D$ is the optical radius and R_D is the disc exponential length-scale; $X = r/R_{opt}$; L is the luminosity. We would like to compare the rotation curve for the color fields (52) with the Universal Rotation Curve (53) where,

for example, $L/L_* = 1$

$$V_{URC} \left(\frac{r}{R_{opt}} \right) = V(R_{opt}) \left[\frac{1.4184 X^{1.22}}{(X^2 + 0.782)^{1.43}} + \frac{1.07251 X^2}{X^2 + 1.5^2} \right]^{1/2} \text{ km s}^{-1}. \quad (54)$$

For the DM the Universal Rotation Curve is

$$V_{DM}^2 \left(\frac{r}{R_{opt}} \right) = V^2(R_{opt}) \frac{1.07251 X^2}{X^2 + 1.5^2} \text{ km}^2 \text{ s}^{-2}. \quad (55)$$

The profiles of $V_{URC}(X)$, $V_{DM}^2(X)$, $V_{LM}^2(X)$ in Fig. 7 are presented (V_{DM}^2 is the rotation curve for the DM, $V_{LM}^2(X)$ is the rotation curve for the light matter).

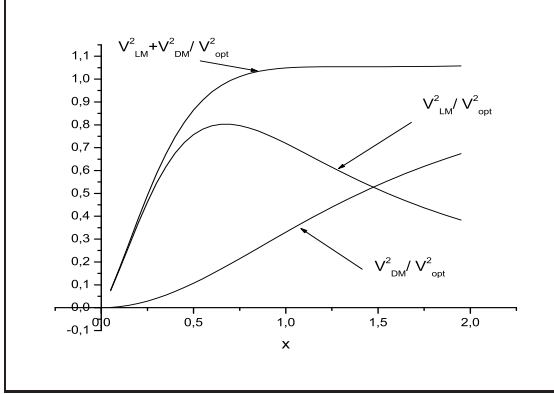


FIG. 7: The profiles of dimensionless rotation curves for the light and dark matter [11].

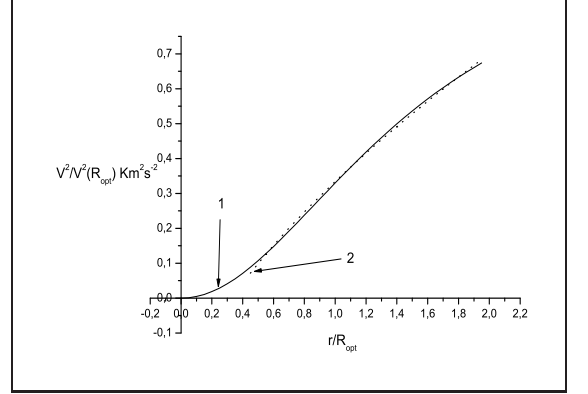


FIG. 8: The comparison of DM rotation curve (55) (curve 1) with the rotation curve (61) (curve 2) for the SU(3) classical color field (A1)-(A4). $\alpha \approx 1.31$, $g' = 1$, $R_{opt} = 20 KPs = 6 * 10^{17} m$, $V_{opt} = 100 K m/s$.

At the center $r \approx 0$ the approximate solution has the form (33) (34) and the mass density (51) approximately is

$$\rho(x) \approx \frac{2}{g^2 c^2 r_0^4} \left(3v_2'^2 + \frac{5}{54} x^2 \right). \quad (56)$$

Consequently the rotation curve will be

$$V^2(r) \approx c^2 \frac{1}{g'^2} \left(\frac{l_{Pl}}{r_0} \right)^2 \left[v_2'^2 \left(\frac{r}{r_0} \right)^2 + \frac{1}{54} w_3'^2 \left(\frac{r}{r_0} \right)^4 \right] \text{ cm}^2 \text{ s}^{-2}. \quad (57)$$

The comparison with Eq. (55) by $x \ll 1$ gives us

$$\left(\frac{r_0}{l_{Pl}} \right)^4 \approx 10^{-6} \left(\frac{c^2}{V^2} \right)^2 \frac{v_2'^2}{g'^2} \left(\frac{R_{opt}}{l_{Pl}} \right)^2. \quad (58)$$

Far away from the center the dimensionless energy density $\varepsilon_\infty(x)$ is presented in Eq. (26) and in this case we can estimate the values of square of speed in the following way

$$\begin{aligned} V^2 &= c^2 \frac{1}{g'^2} \left(\frac{l_{Pl}}{r_0^2} \right)^2 \frac{1}{x} \left(\int_0^x x^2 [\varepsilon(x) - \varepsilon_\infty(x)] dx + \int_{x_1}^x x^2 \varepsilon_\infty(x) dx \right) \\ &\approx \left[c^2 \frac{1}{g'^2} \left(\frac{l_{Pl}}{r_0^2} \right)^2 \frac{1}{x} \int_0^x x^2 \varepsilon_\infty(x) dx \right] - V_0^2, \end{aligned} \quad (59)$$

$$V_0^2 = c^2 \frac{1}{g'^2} \left(\frac{l_{Pl}}{r_0^2} \right)^2 \frac{1}{x} \int_0^x x^2 [\varepsilon_\infty(x) - \varepsilon(x)] dx. \quad (60)$$

The numerical value of V_0^2 is defined near to the center of galaxy where according to Eq. (60) the difference $\varepsilon_\infty(x) - \varepsilon(x)$ is maximal. Thus the asymptotical behavior of the rotation curve for the domain filled with the SU(3) gauge field is

$$V^2 \approx \left[\frac{2}{3} \frac{1}{g'^2} \left(\frac{l_{Pl}}{r_0} \right)^2 \frac{\alpha^2 (\alpha - 1) (3\alpha - 1)}{2\alpha - 1} \left(\frac{r}{r_0} \right)^{2\alpha - 2} \right] c^2 - V_0^2. \quad (61)$$

In Fig. 8 the profiles of the Universal Rotational Curve (55) and fitting curve (61) are presented. The value of parameter α is given from section V A. The details of fitting

$$r_0 \approx 2.01 \cdot 10^{-18} \text{ cm}, \quad (62)$$

$$V_0 \approx 32.25 \text{ Km} \cdot \text{s}^{-1} \quad (63)$$

in Appendix C are presented. One can see that the biggest disagreement is near to the center since close to the center the fitting curve have to be (57) not (61).

C. Transition to non-perturbative quantized phase

The energy density (25) gives us an infinite total mass. How can we avoid this problem? In Section IV we brought forward arguments that the gauge field A_μ^B inside of some region is in classical phase and outside the region is in quantum phase. We think that it is the manifestation of the fact that the gauge field is the strongly interacting field and the quantization of this field should be carried out using a non-perturbative technique. In fact in this paper we would like to show that non-perturbative quantized fields can be spatially distributed in such a way that classical and quantum phases exist simultaneously.

Unfortunately up to now we do not have any exact non-perturbative technique for the quantization. In this section we want to describe briefly approximate non-perturbative technique based on the Heisenberg approach [12] for non-perturbative quantization of a nonlinear spinor field (for details, see [10]).

In section IV we have shown that at some distance from the center the classical phase changes on non-perturbative quantum phase. In Ref. [10] it is shown that two scalar fields may describe a non-perturbative quantized gauge field. Briefly it can be shown by the following way. In quantizing strongly interacting SU(3) gauge fields - via Heisenberg's non-perturbative method [12] one first replaces the classical fields by field operators $\mathcal{A}_\mu^B \rightarrow \hat{\mathcal{A}}_\mu^B$. This yields the following differential equations for the operators

$$\partial_\nu \hat{\mathcal{F}}^{B\mu\nu} = 0. \quad (64)$$

where $\mu, \nu = 0, 1, 2, 3$; $B = 1, 2, \dots, 8$ are SU(3) color indices. These nonlinear equations for the field operators of the nonlinear quantum fields can be used to determine expectation values for the field operators $\hat{\mathcal{A}}_\mu^B$. One problem in using these equations in order to obtain expectation values like $\langle \mathcal{A}_\mu^B \rangle$, is that these equations involve not only powers or derivatives of $\langle \mathcal{A}_\mu^B \rangle$ (*i.e.* terms like $\partial_\alpha \langle \mathcal{A}_\mu^B \rangle$ or $\partial_\alpha \partial_\beta \langle \mathcal{A}_\mu^B \rangle$) but also contain terms like $\mathcal{G}_{\mu\nu}^{BC} = \langle \mathcal{A}_\mu^B \mathcal{A}_\nu^C \rangle$. Starting with Eq. (64) one can generate an operator differential equation for the product $\hat{\mathcal{A}}_\mu^B \hat{\mathcal{A}}_\nu^C$ consequently allowing the determination of the Green's function $\mathcal{G}_{\mu\nu}^{BC}$

$$\langle Q | \hat{\mathcal{A}}^B(x) \partial_{y\nu} \hat{\mathcal{F}}^{B\mu\nu}(x) | Q \rangle = 0. \quad (65)$$

However this equation will in its turn contain other, higher order Green's functions. Repeating these steps leads to an infinite set of equations connecting Green's functions of ever increasing order. This construction, leading to an infinite set of coupled, differential equations, does not have an exact, analytical solution and so must be handled using some approximation. The basic approach in this case is to give some physically reasonable scheme for cutting off the infinite set of equations for the Green's functions. Using some assumptions and approximations on 2- and 4-points Green's functions one can reduce the initial SU(3) Lagrangian to an effective Lagrangian describing two interacting scalar fields (for details see Ref. [10]). The scalar fields ϕ and χ which are under discussion here appear in the following way. We assume that in the first approximation two points Green's functions can be calculated as follows

$$\langle A_i^a(x) A_j^b(y) \rangle = -\eta_{ij} f^{apm} f^{bpn} \chi^m(x) \chi^n(y), \quad (66)$$

$$\langle A_0^a(x) A_0^b(y) \rangle \ll \langle A_i^a(x) A_j^b(y) \rangle \quad (67)$$

where $A_\mu^a \in SU(2) \subset SU(3)$, $a = 1, 2, 3$; $m = 4, 5, 6, 7, 8$; $i, j = 1, 2, 3$ are spatial indices. And

$$\langle A_i^m(x) A_j^n(y) \rangle = -\eta_{ij} f^{mpa} f^{npb} \phi^a(x) \phi^b(y), \quad (68)$$

$$\langle A_0^m(x) A_0^n(y) \rangle \ll \langle A_i^m(x) A_j^n(y) \rangle \quad (69)$$

where $A_\mu^m \in SU(3)/SU(2)$. The 4-points Green's functions are a bilinear combination of 2-points Green's functions

$$\begin{aligned} \langle A_\mu^m(x) A_\nu^n(y) A_\alpha^p(z) A_\beta^q(u) \rangle &= \lambda_1 \left[\langle A_\mu^m(x) A_\nu^n(y) \rangle \langle A_\alpha^p(z) A_\beta^q(u) \rangle + \right. \\ &\left. \frac{\mu_1^2}{4} \left(\delta^{mn} \eta_{\mu\nu} \langle A_\alpha^p(z) A_\beta^q(u) \rangle + \delta^{pq} \eta_{\alpha\beta} \langle A_\mu^m(x) A_\nu^n(y) \rangle \right) + \frac{\mu_1^4}{16} \delta^{mn} \eta_{\mu\nu} \delta^{pq} \eta_{\alpha\beta} \right] + \\ &\text{(permutations of indices)} \end{aligned} \quad (70)$$

and

$$\begin{aligned} \langle A_\mu^a(x) A_\nu^b(y) A_\alpha^c(z) A_\beta^d(u) \rangle &= \lambda_2 \left[\langle A_\mu^a(x) A_\nu^b(y) \rangle \langle A_\alpha^c(z) A_\beta^d(u) \rangle + \right. \\ &\left. \frac{\mu_2^2}{4} \left(\delta^{ab} \eta_{\mu\nu} \langle A_\alpha^c(z) A_\beta^d(u) \rangle + \delta^{cd} \eta_{\alpha\beta} \langle A_\mu^a(x) A_\nu^b(y) \rangle \right) + \frac{\mu_2^4}{16} \delta^{ab} \eta_{\mu\nu} \delta^{cd} \eta_{\alpha\beta} \right] + \\ &\text{(permutations of indices)} \end{aligned} \quad (71)$$

here $\lambda_{1,2}, \mu_{1,2}$ are some constants. The assumptions (66)-(71) allows us to average the SU(3) Lagrangian

$$\mathcal{L}_{SU(3)} = -\frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu}, \quad A = 1, 2, \dots, 8 \quad (72)$$

and bring it to the form

$$\mathcal{L}_{eff} = \langle \mathcal{L}_{SU(3)} \rangle = \frac{1}{2} (\partial_\mu \phi^a) (\partial^\mu \phi^a) + \frac{1}{2} (\partial_\mu \chi^m) (\partial^\mu \chi^m) - V(\phi^a, \chi^m), \quad (73)$$

$$V(\phi^a, \chi^m) = \frac{\lambda_1}{4} (\phi^a \phi^a - \mu_1^2)^2 - \frac{\lambda_2}{4} (\chi^m \chi^m - \mu_2^2)^2 - \frac{1}{2} (\phi^a \phi^a) (\chi^m \chi^m) \quad (74)$$

with the field equations

$$\nabla_\mu (\nabla^\mu \phi^a) = -\frac{\partial V(\phi^a, \chi^m)}{\partial \phi^a}, \quad (75)$$

$$\nabla_\mu (\nabla^\mu \chi^m) = -\frac{\partial V(\phi^a, \chi^m)}{\partial \chi^m}. \quad (76)$$

Let us consider the spherically symmetric case $\phi^a = k\phi(r)$, $\chi^m = k\chi(r)$ where k is some constant. In this case the field equations are

$$\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = \phi [\chi^2 + \lambda_1 (\phi^2 - \mu_1^2)], \quad (77)$$

$$\frac{d^2 \chi}{dr^2} + \frac{2}{r} \frac{d\chi}{dr} = \chi [\phi^2 + \lambda_2 (\chi^2 - \mu_2^2)]. \quad (78)$$

It is easy to see that asymptotically the solution has the form

$$\phi(x) \approx m_1 + \phi_\infty \frac{e^{-(r-r_q)\sqrt{2\lambda_1\mu_1^2}}}{r}, \quad (79)$$

$$\chi(x) \approx \chi_\infty \frac{e^{-(r-r_q)\sqrt{m_1^2 - \lambda_2\mu_2^2}}}{r} \quad (80)$$

where $\phi_\infty, \chi_\infty, r_q$ are constants. We think that this solution describes the non-perturbative quantized SU(3) gauge field after the transition from classical phase to quantum one occurs.

The main point of this consideration is that the non-perturbative quantized gauge field decreases very quickly (exponentially) after transition to the quantum phase and consequently the total mass becomes finite one.

VI. INVISIBILITY OF COLORED DARK MATTER

For the detection of DM (in the context of the model presented here) we can use only *colored* particles which can interact with color gauge fields. The equations describing the motion of a colored particle are Wong's equations

$$m \frac{d^2 x^\mu}{ds^2} = -g F_\nu^{A\mu} Q^A \frac{dx^\nu}{ds}, \quad (81)$$

$$\frac{dQ^A}{ds} = -gf^{ABC} \left(A_\mu^B \frac{dx^\mu}{ds} \right) Q^C \quad (82)$$

where $x^\mu(s)$ is the 4D trajectory of the particle with the mass m , Q^A is the color components of color charge of the particle, $(Q^A)^2 = Q^2 = \text{const.}$ The ordinary elementary particles are colorless and consequently do not interact with the color DM.

Only 't-Hooft - Polyakov monopoles and dyons may interact with colored DM and can be used for the detection of the colored DM. Another possibility of the interaction of an elementary particle with colored DM is the interaction between external color field (DM) and a color electric and/or magnetic dipole or quadrupole of this elementary particle.

VII. CONCLUSIONS

In this paper we have considered the idea that the problem of DM probably can be connected with the problem of non-perturbative quantization of strongly interacting fields. It allows us to connect one problem in macroscopical physics (the problem of DM) with another problem in microscopical physics (confinement problem on quantum chromodynamics). In this connection R. Kolb in Ref. [13] write: "Dark matter and dark energy are two of the binding cords I will use to illustrate how collaborations of astronomers and high energy physicists on large astronomical projects can be good for astronomy, and how discoveries in astronomy can guide high-energy physicists in their quest for understanding nature on the smallest scales."

The features of the model of colored DM presented here are:

- The estimation of the parameter α from the Yang-Mills equations (see Appendix B) and from the astrophysical (see Appendix C) point of view is in agreement with remarkable accuracy.
- On the background of a non-perturbative vacuum of non-Abelian gauge field there exists a bubble of the same classical gauge field. The classical non-Abelian gauge field filled the bubble is colored DM.
- Spherically symmetric classical solutions of the Yang-Mills equations have weakly decreasing mass density distribution leading to a good agreement with the Universal Rotation Curve.
- The distribution of these classical gauge field is that on some distance from the center the transition from classical phase to quantum occurs.

The problems for the future investigations are:

- The theoretical estimation of V_0^2 parameter and comparing it with the fitted value (63).
- The fitting of the Universal Rotational Curve (55) using the function joining (57) and (61).
- The most important problem in the model presented here is the calculation of gauge field distribution using a non-perturbative quantization technique.
- The search for possibility of the classical gauge fields detection .

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APPENDIX A: ANSATZ FOR SU(3) GAUGE POTENTIAL IN MINKOWSKI SPACETIME

We consider the classical SU(3) Yang-Mills gauge field A_μ^B and use the following ansatz for the $SU(2) \in SU(3)$ components of the gauge field [9]

$$A_0^2 = -2 \frac{z}{gr^2} \chi(r), \quad A_0^5 = 2 \frac{y}{gr^2} \chi(r), \quad A_0^7 = -2 \frac{x}{gr^2} \chi(r), \quad (A1)$$

$$A_i^2 = 2 \frac{\epsilon_{3ij} x^j}{gr^2} [h(r) + 1], \quad (A2)$$

$$A_i^5 = -2 \frac{\epsilon_{2ij} x^j}{gr^2} [h(r) + 1], \quad (A3)$$

$$A_i^7 = 2 \frac{\epsilon_{1ij} x^j}{gr^2} [h(r) + 1] \quad (A4)$$

where $F_{\mu\nu}^B = \partial_\mu A_\nu^B - \partial_\nu A_\mu^B + gf^{ABC} A_\mu^B A_\nu^C$ is the field strength tensor; f^{ABC} are the SU(3) structural constants; $A, B, C = 1, 2, \dots, 8$ are color indices; g is the coupling constant and

$$(A_0)_{\alpha\beta} = 2 \left(\frac{x^\alpha x^\beta}{r^2} - \frac{1}{3} \delta^{\alpha\beta} \right) \frac{w(r)}{gr}, \quad (A5)$$

$$(A_i)_{\alpha\beta} = 2 (\epsilon_{is\alpha} x^\beta + \epsilon_{is\beta} x^\alpha) \frac{x^s}{gr^3} v(r), \quad (A6)$$

for the coset components belonging to the coset space $SU(3)/SU(2)$; $i = 1, 2, 3$ are space indices; ϵ_{ijk} is the absolutely antisymmetric Levi-Civita tensor; the functions $\chi(r), h(r), w(r), v(r)$ are unknown functions. The coset components $(A_\mu)_{\alpha\beta}$ in the matrix form are written as

$$(A_\mu)_{\alpha\beta} = \sum_{a=1,3,4,6,8} A_\mu^B (T^B)_{\alpha\beta} \quad (A7)$$

where $T^B = \frac{\lambda^B}{2}$ are the SU(3) generators, λ^B are the Gell-Mann matrices.

This ansatz in the spherical coordinate system is

$$A_t^a = \left\{ w(r) \sin^2 \theta \sin(2\varphi); \quad -2\phi(r) \cos \theta; \quad w(r) \sin^2 \theta \cos(2\varphi); \quad w(r) \sin(2\theta) \cos \phi; \right. \\ \left. 2\phi(r) \sin \theta \sin \varphi; \quad w(r) \sin(2\theta) \sin \phi; \quad -\phi(r) \sin(2\theta); \quad w(r) \frac{1 + 3 \cos(2\theta)}{2\sqrt{3}} \right\}; \quad (A8)$$

$$A_r^a = 0; \quad (A9)$$

$$A_\theta^a = \left\{ -2v(r) \cos(2\varphi) \sin \theta; \quad 0; \quad 2v(r) \sin \theta \sin(2\varphi); \quad 2v(r) \cos \theta \sin \varphi; \quad 2[1 + h(r)] \cos \varphi; \right. \\ \left. -2v(r) \cos \theta \cos \varphi; \quad 2[1 + h(r)] \sin \varphi; \quad 0 \right\}; \quad (A10)$$

$$A_\varphi^a = \left\{ v(r) \sin(2\theta) \sin(2\varphi); \quad -2[1 + h(r)] \sin \theta; \quad v(r) \sin(2\theta) \cos(2\varphi); \quad 2v(r) \cos(2\theta) \cos \varphi; \right. \\ \left. -2[1 + h(r)] \cos \theta \sin \varphi; \quad 2v(r) \cos(2\theta) \sin \varphi; \quad 2[1 + h(r)] \cos \theta \cos \varphi; \quad \sqrt{3}v(r) \sin(2\theta) \right\}. \quad (A11)$$

APPENDIX B: FITTING OF PARAMETER α

For the estimation of the parameter α we use the functions $w(x)$ and $\varepsilon(x)$ given from the numerical solution of Eq's (20) (21) (see also Fig. 5) in the region $20 \leq x \leq 100$. The fitting functions are the asymptotical form of the function $w(x)$ (see Eq. (23))

$$w(x) \approx \alpha x^\alpha + w_0 \quad (B1)$$

and for the control we use the asymptotical form of the function $\varepsilon(x)$ (see Eq. (26))

$$\varepsilon_\infty(x) \approx \frac{2}{3} \alpha^2 (\alpha - 1) (3\alpha - 1) x^{2\alpha-4} + \varepsilon_0 \quad (B2)$$

here w_0 and ε_0 are systematical errors in the consequence of ignoring of other components in the asymptotical decompositions (B1) and (B2).

The fitting carried out using MATHEMATICA package. The fitting parameters are: α and either w_0 or ε_0 . The result of fitting is

$$\alpha \approx 1.31077, \quad w_0 \approx 2.15369 \quad (\text{B3})$$

for fitting $w(x)$ and

$$\alpha \approx 1.31995, \quad \varepsilon_0 \approx -0.0012265 \quad (\text{B4})$$

for fitting $\varepsilon(x)$.

APPENDIX C: FITTING OF ROTATIONAL CURVE OF GAUGE FIELD

For the fitting of the rotational curve (61) we use the data from the Universal Rotational Curve (55). The fitting equation is equation (61) in the form

$$\frac{V^2}{V_{opt}^2} = Ax^B + C, \quad (\text{C1})$$

$$A = \frac{2}{3} \frac{\alpha^2 (\alpha - 1) (3\alpha - 1)}{2\alpha - 1} \frac{c^2}{V_{opt}^2} \frac{1}{g'^2} \left(\frac{l_{Pl}}{r_0} \right)^2 \left(\frac{R_{opt}}{r_0} \right)^{2\alpha-2} \quad (\text{C2})$$

$$B = 2\alpha - 2, \quad (\text{C3})$$

$$C = -V_0^2 \quad (\text{C4})$$

where the fitted parameters are A, B, C . The fitting is carried out using MATHEMATICA package. The fitting parameters are: r_0 and V_0^2 . The result of fitting is

$$\alpha = 1.31954, \quad r_0 \approx 2.01 \cdot 10^{-18} \text{ cm}, \quad V_0 \approx 32.25 \text{ Km} \cdot \text{s}^{-1}. \quad (\text{C5})$$

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